Signal Analysis & Communication ECE 355

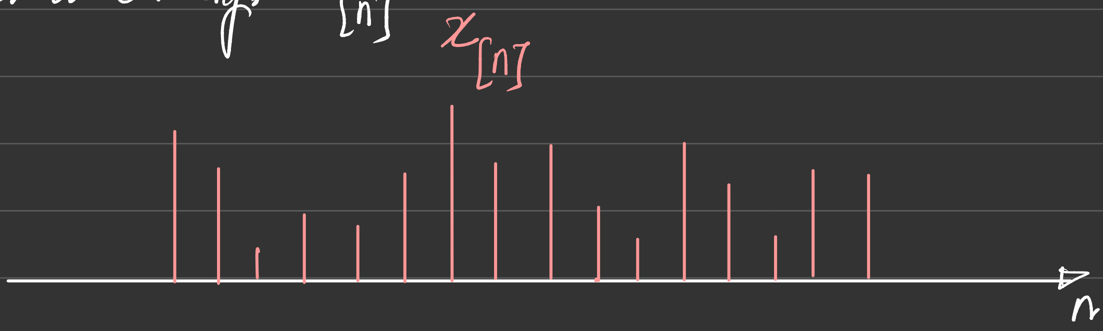
Ch. 7.5. Sampling of DT Signals

Lecture 31

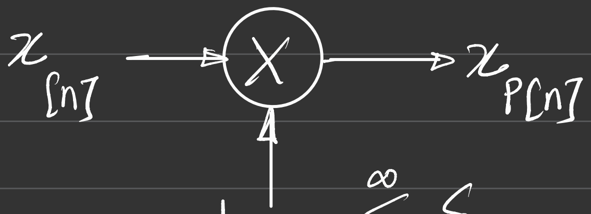
27-11-2023

Ch. 7.5. Sampling of Discrete Time Signals

- Consider a DT sig. $x_{[n]}$



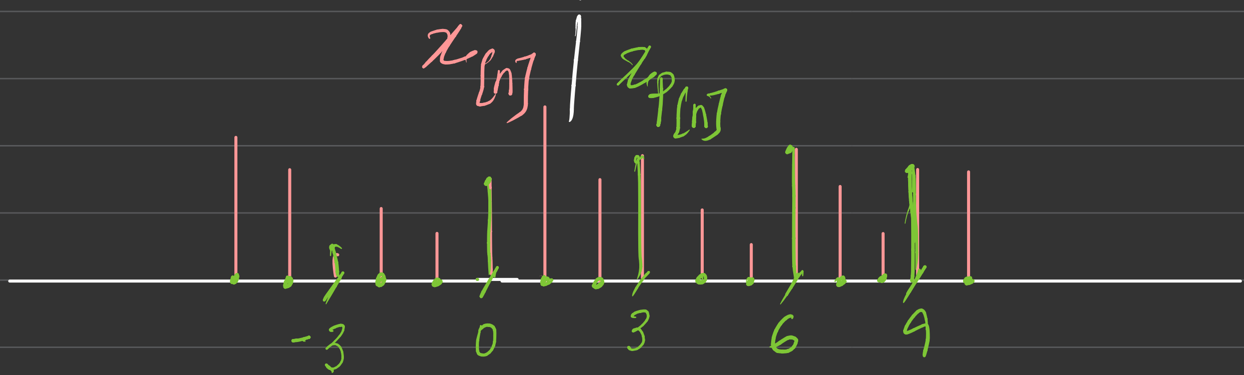
- In analogy with CT Sampling, sampling of DT sig. can be performed as:



$$p_{[n]} = \sum_{k=-\infty}^{\infty} \delta_{[n-kN]}$$

- Sampling period $= N$

- For example, the above sig. for $N=3$ would be sampled as:



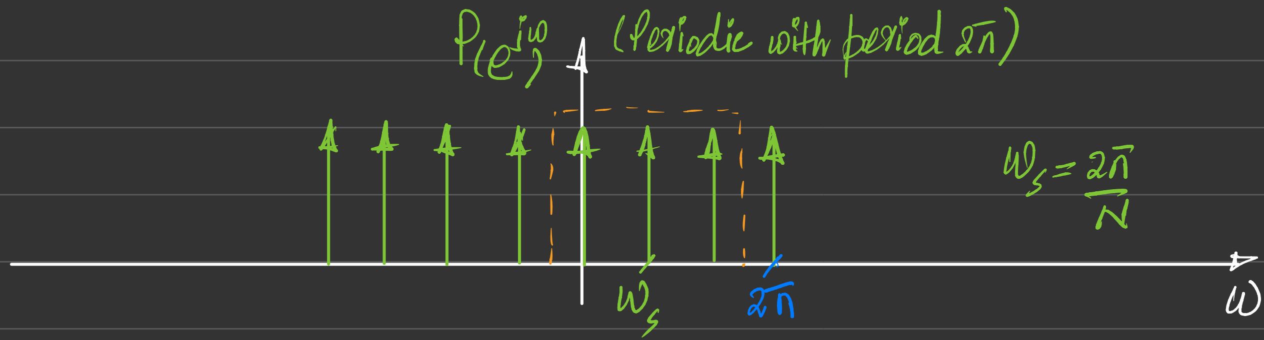
$$x_{p[n]} = \sum_{k=-\infty}^{\infty} x_{[kN]} \delta_{[n-kN]} \quad — (1)$$

$$= \begin{cases} x_{[n]} \,, & \text{if } n = kN \text{ for some } k \in \mathbb{Z} \\ 0 \,, & \text{o/w} \end{cases}$$

- Previously, the DTFT of DT impulse-train is (Ref. Lec. 24)

$$P(e^{j\omega}) = \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

Periodic with period 2π.



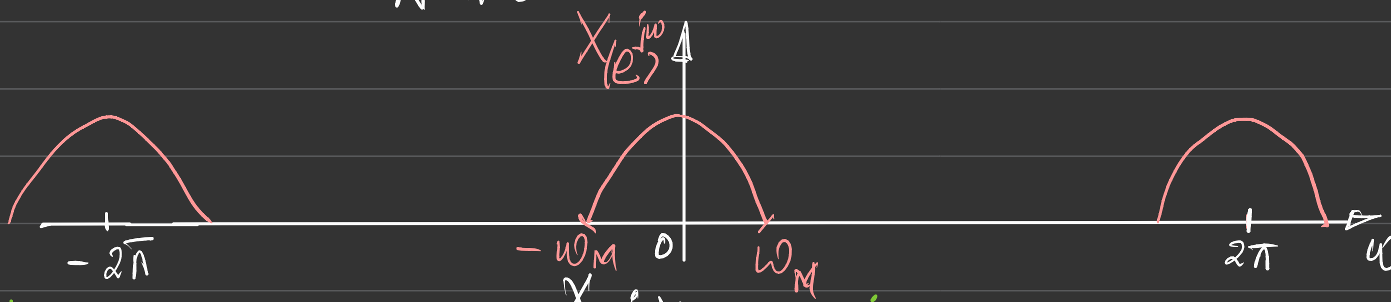
- Accordingly, the spectra of the sampled sig. is given as: (DTFT of eqn (1) - multiplication in time domain is convolution in freq.-domain)

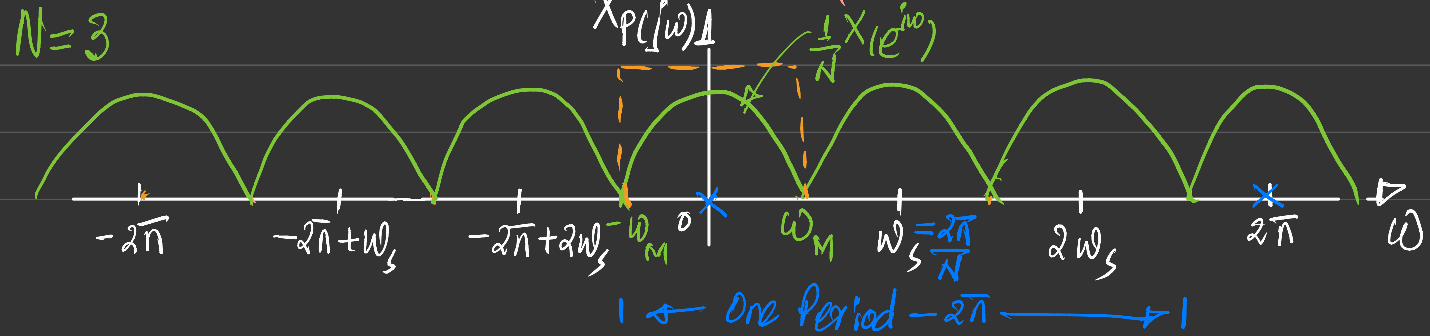
$$X_p(j\omega) = \frac{1}{2\pi} \int_{2\pi} X(e^{j(\omega-\theta)})\, P(e^{j\theta})\, d\theta$$

since the integral is from $0 \to 2\pi$

$$= \frac{1}{\cancel{2\pi}} \int_{0^-}^{2\pi^-} X(e^{j(\omega-\theta)})\, \frac{\cancel{2\pi}}{N} \sum_{k=0}^{N-1} \underbrace{\delta(\theta - k\omega_s)}_{\text{exists at } \theta = k\omega_s} d\theta$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\omega - k\omega_s)})$$





- Again, no aliasing if $\omega_s > 2\omega_M$.

- Reconstruction by ideal lowpass filtering.

$$H(e^{j\omega}) = \begin{cases} N, & |\omega| < \omega_c, \\ 0, & o/w \end{cases} \quad \text{Periodic (as is discrete in time)}$$

$$\& \quad \omega_M < \omega_c < \omega_s - \omega_M$$

$$\text{Usually } \omega_c = \frac{\omega_M}{2}$$

- The corresponding time-domain sig. is:

$$h[n] = N\frac{\omega_c}{\pi} \frac{\sin(\omega_c n)}{\omega_c n}$$

- Therefore, the reconstructed sig. after filtering is:

$$x_r[n] = x_p[n] * h[n]$$

$$= \sum_{m=-\infty}^{\infty} x_p[m]\, h[n-m]$$

'Conv. in DT'

$$= \sum_{k=-\infty}^{\infty} x_p[kN]\, N\frac{\omega_c}{\pi} \frac{\sin(\omega_c(n-kN))}{\omega_c(n-kN)}$$

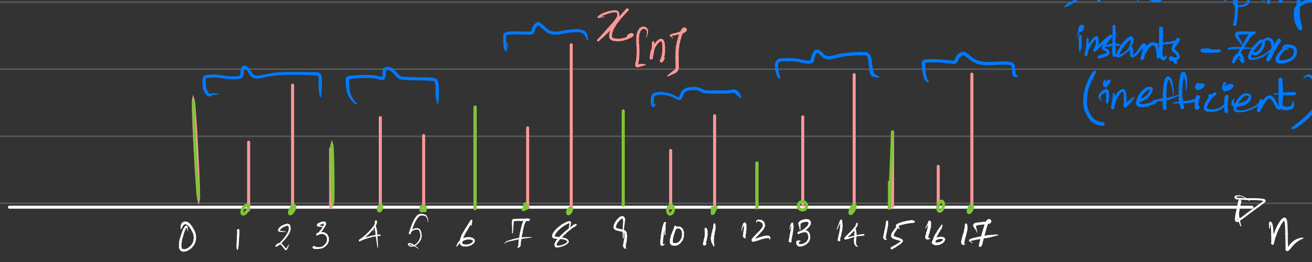
- DOWNSAMPLING ("Decimation")

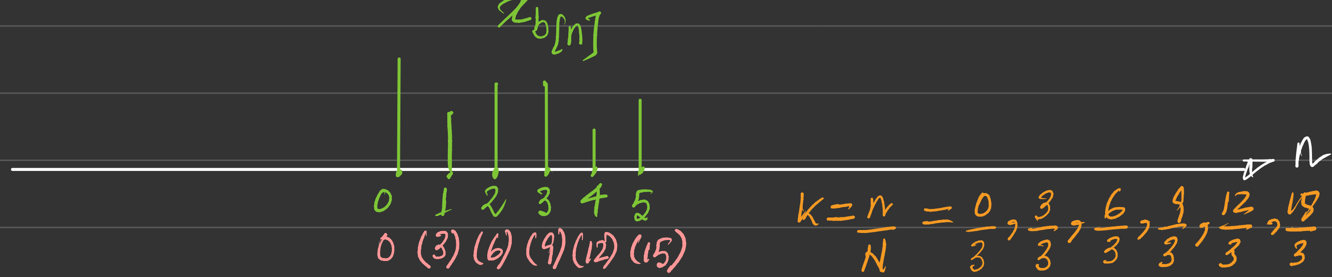
- In many applications of DT sampling, such as filter design & implementation or in communication, it is inefficient to represent, transmit, or store the sampled sequence $x_p[n]$ directly, since in b/w the sampling instants, $x_p[n]$ is known to be zero.

- Thus, the sampled sequence is typically replaced by a new seq. $x_{b[n]}$, which is simply every Nth value of $x_{p[n]}$; that is,

$$\boxed{x_{b[n]} = x_{p[nN]} = x_{[nN]}}$$

In b/w sampling instants - zero (inefficient)





$$K = \frac{n}{N} = \frac{0}{3}, \frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}, \frac{15}{3}$$

$$0, 1, 2, 3, 4, 5$$

- The operation of extracting every Nth sample is commonly referred to as "decimation".

- To determine the effect of decimation in freq. domain, let's determine the relationship b/w $X_{b(e^{j\omega})}$ & $X_{(e^{j\omega})}$.

$$X_{b(e^{j\omega})} = \sum_{k=-\infty}^{\infty} x_{b[k]} e^{-jk\omega}$$

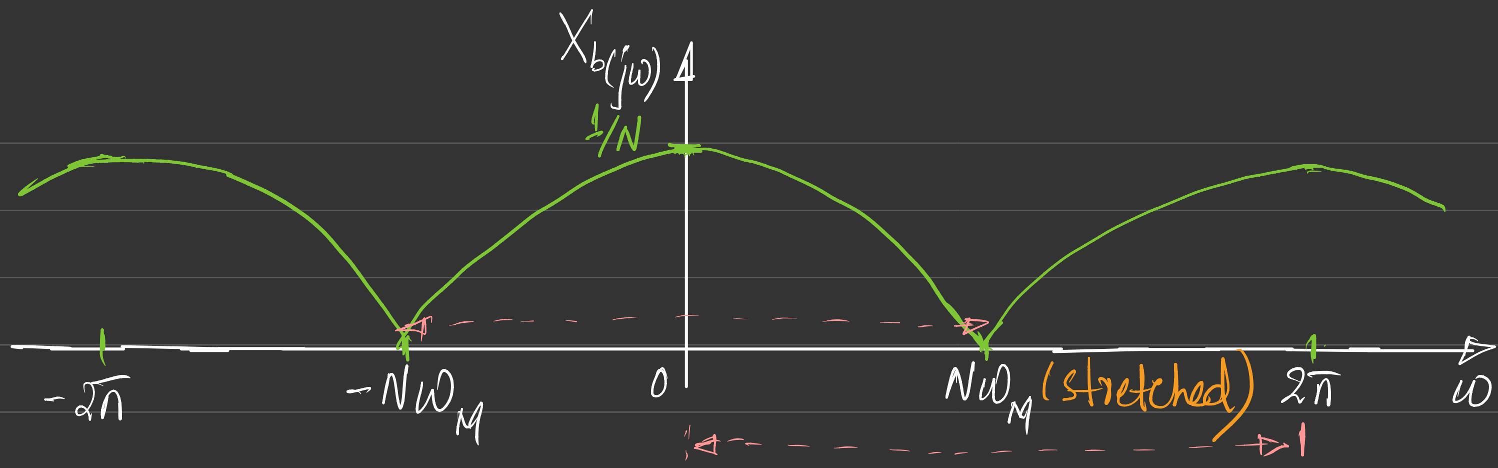
$$= \sum_{n=-\infty}^{\infty} x_{p[n]} e^{-j\omega \frac{n}{N}}$$

replace 'k' by 'n'

$$= X_p(e^{-j\omega/N})$$

Stretch $X_{P(j\omega)}$ by a factor of 'N'.

$$\omega_s \xrightarrow{\text{stretched to}} 2\pi$$



- If no aliasing in $X_p(e^{j\omega})$, then no loss of information in downsampling.

UPSAMPLING ("Interpolation")

- For some applications, it is useful to convert a sequence to a higher equivalent sampling rate, referred to as "interpolation".
- Reverse of "Decimation".

$$x_b[n] \xrightarrow{\text{Stuff zeros}} x_p[n] \xrightarrow{\text{LPF}} x_r[n]$$

Reconstructed.

- If no aliasing, then $x_r[n] = x[n]$

RESAMPLING

- Upsample then downsample.
- Example:
  - Convert 800x600 image to 1280x960 (1.6 times)

    Not integer.

- Therefore we first upsample by 8, then downsample by 5

$$\frac{8}{5} = 1.6 \checkmark$$